Modified teleparallel theories of gravity

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Outline

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1. Introduction
   - Teleparallel gravity
   - $f(R)$ and $f(T)$ gravity
2. Modified teleparallel theories of gravity $f(T, B)$
3. Gauss-Bonnet and trace extensions $f(T, B, T_G, B_G, \mathcal{T})$
   - Gauss Bonnet extension
   - Trace of the energy-momentum tensor extension
4. Conclusions

Teleparallel equivalent of general relativity

- A general connection which defines the parallel transportation is

Most general spin connection (Einstein-Cartan theory)

$$\overbrace{w_{\mu}{}^{\lambda}{}_{\nu}}^{\text{General spin connection}} = \overbrace{\Gamma^{\lambda}_{\mu\nu}}^{\text{related with curvature}} + \overbrace{\tilde{K}_{\mu}{}^{\lambda}{}_{\nu}}^{\text{related with torsion}} \,.$$

- G.R. assumes for simplicity that $\tilde{K}_{\mu}{}^{\lambda}{}_{\nu} \equiv 0$
- Weitzenböck noticed that it is always possible to define a connection $W_{\mu}{}^{\lambda}{}_{\nu}$ on a space such that is globally flat $(R^{\lambda}{}_{\mu\nu\sigma} \equiv 0) \Longrightarrow$ TEGR

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Teleparallel equivalent of general relativity

- The teleparallel action is formulated based on a gravitational scalar called the torsion scalar T

TEGR action

$$S_{\text{TEGR}} = \int \left[-\frac{T}{2\kappa} + L_{\text{m}} \right] e \, d^4x \, .$$

Here, $e = \det(e^{\mu}_{a}) = \sqrt{-g}$ and $\kappa = 8\pi G$.

- The relationship between the scalar curvature R and the scalar torsion is

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- A well studied modification of GR is $f(R)$ gravity, which has the following action

$f(R)$ gravity action

$$S_{f(R)} = \int f(R)\sqrt{-g}\, d^4x \,.$$

- Here, f is an arbitrary (sufficiently smooth) function of the Ricci scalar.
- Ricci scalar depends on second derivatives of the metric tensor $\rightarrow$ **Fourth order theory.**

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$f(T)$ gravity

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$f(T)$ gravity action

$$S_{f(T)} = \int f(T) e \, d^4x \, .$$

- The torsion scalar T depends on the first derivatives of the tetrads $\rightarrow$ **Second order theory.**
- T is not invariant under local LT $\Longrightarrow$ $f(T)$ **is also not invariant under local LT.**

Not equivalency between $f(R)$ and $f(T)$

Field equations of $f(T) \neq$ Field equations of $f(R)$

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$f(T, B)$ gravity

- Inspired by the later discussion, we define the action (Bahamonde et. al Phys.Rev. D92,10, 104042 (2015))

$f(T, B)$ gravity action

$$S_{f(T,B)} = \int \left[\frac{1}{\kappa} f(T, B) + L_{\rm m} \right] e\, d^4x \,,$$

where f is a function of both of its arguments and $L_{\rm m}$ is a matter Lagrangian.

- Since $R = -T + B$, by setting $f = f(-T + B) = f(R)$ we recover $f(R)$ gravity.

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Gauss-Bonnet extension

- The Gauss-Bonnet term is a quadratic combination of the Riemann tensor and its contractions given by

Gauss-Bonnet term

$$G = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\kappa\lambda}R^{\mu\nu\kappa\lambda}\,.$$

- This term can be expressed in a fashion similar form as before which simply reads

Relationship between Gauss-Bonnet G and T_G

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Gauss-Bonnet extension action

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Gauss-Bonnet extension action

$$S_{f(T,B,T_G,B_G)} = \int \left[\frac{1}{2\kappa} f(T,B,T_G,B_G) + L_{\rm m}\right] e\, d^4x\,.$$

- This action is very general. For instance, we can recover $f = f(T,T_G)$ gravity or $f(-T+B,-T_G+B_G) = f(R,G)$ gravity.

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Trace extension

We will now consider the later framework and include the trace of the energy-momentum tensor to the action discussed before. This gives the extended action

Trace and Gauss-Bonnet action extension

$$S_{f(T,B,T_G,B_G,\mathcal{T})} = \int \left[\frac{1}{2\kappa} f(T, B, T_G, B_G, \mathcal{T}) + L_{\mathrm{m}}\right] e\, d^4x \,,$$

where additionally f is a function of the trace of the energy-momentum tensor $\mathcal{T} = E_a^{\beta} \mathcal{T}_{\beta}^{a}$

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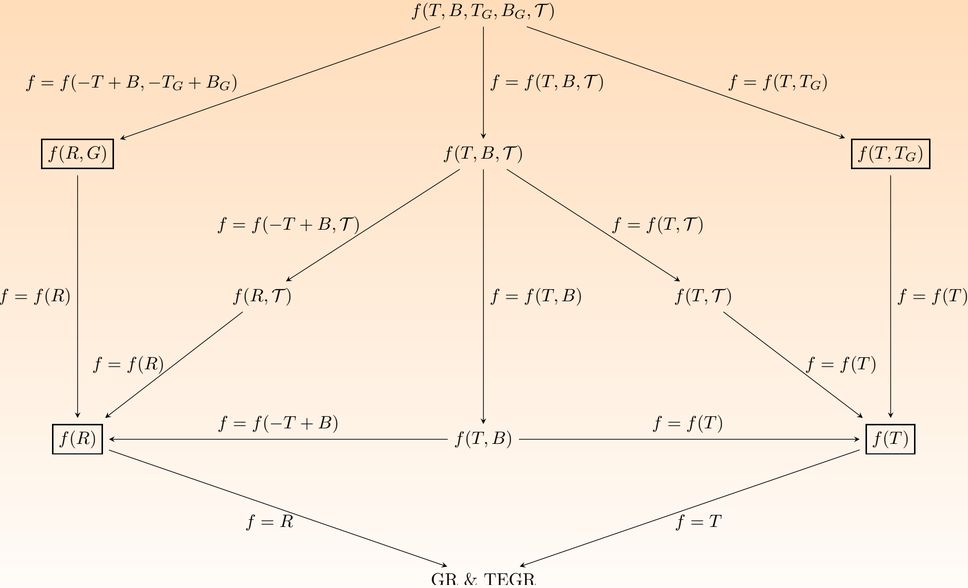
$f(T, B, T_G, B_G, \mathcal{T})$ diagram



Conclusions

- For many years now, an ever increasing number of modifications of GR has been considered. Many of these theories were considered in isolation in the past and their relationship with other similarly looking theories was only made implicitly.
- In our works, we explicitly found the corresponding teleparallel equivalent to well-known theories of gravity such as $f(R)$, $f(R,G)$, $f(R,\mathcal{T})$ and others.

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