QUANTUM STRUCTURE OF SPACETIME AND GRAVITY

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MAXWELL-WEYL GAUGE THEORY OF GRAVITY

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MOTIVATION

1. Maxwell group
2. Summary of Literature
3. Influence area of Maxwell group

What is the Maxwell Algebra?

- The Maxwell algebra is a non-central extension of the Poincare algebra, in which the momentum generators no longer commute, but satisfy $[P_a, P_b] = Z_{ab}$. The charges Z_{ab} commute with the momenta, and transform tensorially under the action of the angular momentum generators.
- If one constructs an action for a massive particle, invariant under these symmetries, one finds that it satisfies the equations of motion of a charged particle interacting with a constant electromagnetic field via the Lorentz force.

G.W. Gibbons, J. Gomis and C. N. Pope, PHYSICAL REVIEW D 82, 065002 (2010)

$$[P_a, P_b] = 0 \quad \rightarrow \quad [\tilde{P}_a, \tilde{P}_b] = iZ_{ab}$$

Maxwell Algebra

- If we enlarge the Poincare algebra by six additional tensorial abelian generators as showed below then we get Maxwell algebra.

Maxwell algebra

$$[M_{ab}, M_{cd}] = i(\eta_{ad}M_{bc} + \eta_{bc}M_{ad} - \eta_{ac}M_{bd} - \eta_{bd}M_{ac})$$

$$[M_{ab}, \Pi_c] = i(\eta_{bc}\Pi_a - \eta_{ac}\Pi_b)$$

$$[\Pi_a, \Pi_b] = iF_{ab}$$

$$[M_{ab}, F_{cd}] = i(\eta_{ad}F_{bc} + \eta_{bc}F_{ad} - \eta_{ac}F_{bd} - \eta_{bd}F_{ac})$$

$$[F_{ab}, F_{cd}] = 0$$

$$[F_{ab}, \Pi_c] = 0$$

New momentum op.

$$\Pi_a = i(\partial_a + eA_a(x)) \Rightarrow (i\gamma^a\Pi_a - m)\psi(x,f) = 0$$

$$A_a(x) = \tfrac{1}{2} f_a{}^b x_b \Rightarrow f_{ab} = \partial_a A_b(x) - \partial_b A_a(x)$$

Where,
A_a: Electromagnetic potential, e: Electric charge
f_{ab}: Electromagnetic field tensor

This antisymmetric generator $F_{[ab]}$ can be used to describe the motion of a relativistic particle in a constant electromagnetic field.

If we select $F_{[ab]} = 0$ then we get well-known Poincare algebra which describes flat Minkowski space-time.

H. Bacry, P. Combe, and J.L. Richard, Nuovo Cim. 67, (1970), 267
R. Schrader, Fortsch.Phys. 20, (1972), 701-734

Maxwell Algebra

- In 2005 D. V. Soroka, V. A. Soroka shows us the Maxwell algebra can be found with new additional tensorial coordinate θ^{ab} and corresponding tensorial derivative $\partial_{ab} = \frac{\partial}{\partial \theta^{ab}}$.

Maxwell group

$$[M_{ab}, M_{cd}] = i(\eta_{ad}M_{bc} + \eta_{bc}M_{ad} - \eta_{ac}M_{bd} - \eta_{bd}M_{ac})$$

$$[M_{ab}, P_c] = i(\eta_{bc}P_a - \eta_{ac}P_b)$$

$$[P_a, P_b] = iZ_{ab}$$

$$[M_{ab}, Z_{cd}] = i(\eta_{ad}Z_{bc} + \eta_{bc}Z_{ad} - \eta_{ac}Z_{bd} - \eta_{bd}Z_{ac})$$

$$[Z_{ab}, Z_{cd}] = 0$$

$$[Z_{ab}, P_c] = 0$$

Differential realisation of generators:

$$P_a = i(\partial_a - \tfrac{1}{2}x^b\partial_{ab})$$

$$Z_{ab} = i\partial_{ab}$$

$$M_{ab} = i(x_a\partial_b - x_b\partial_a) + 2i(\theta_a{}^c\partial_{bc} - \theta_b{}^c\partial_{ac})$$

D. V. Soroka, V. A. Soroka, Physics Letters B 607 (2005) 302–305.

Some relations of tensorial coordinates and its derivatives:

$$\partial_{ab}\theta^{cd} = \frac{1}{2}\left(\delta_a^c\delta_b^d - \delta_b^c\delta_a^d\right)$$

$$\partial_{ab}x^c = 0$$

$$[\partial_{ab}, \partial_c] = 0$$

$$[\partial_{ab}, \partial_{cd}] = 0$$

A Brief Summary of Litarature

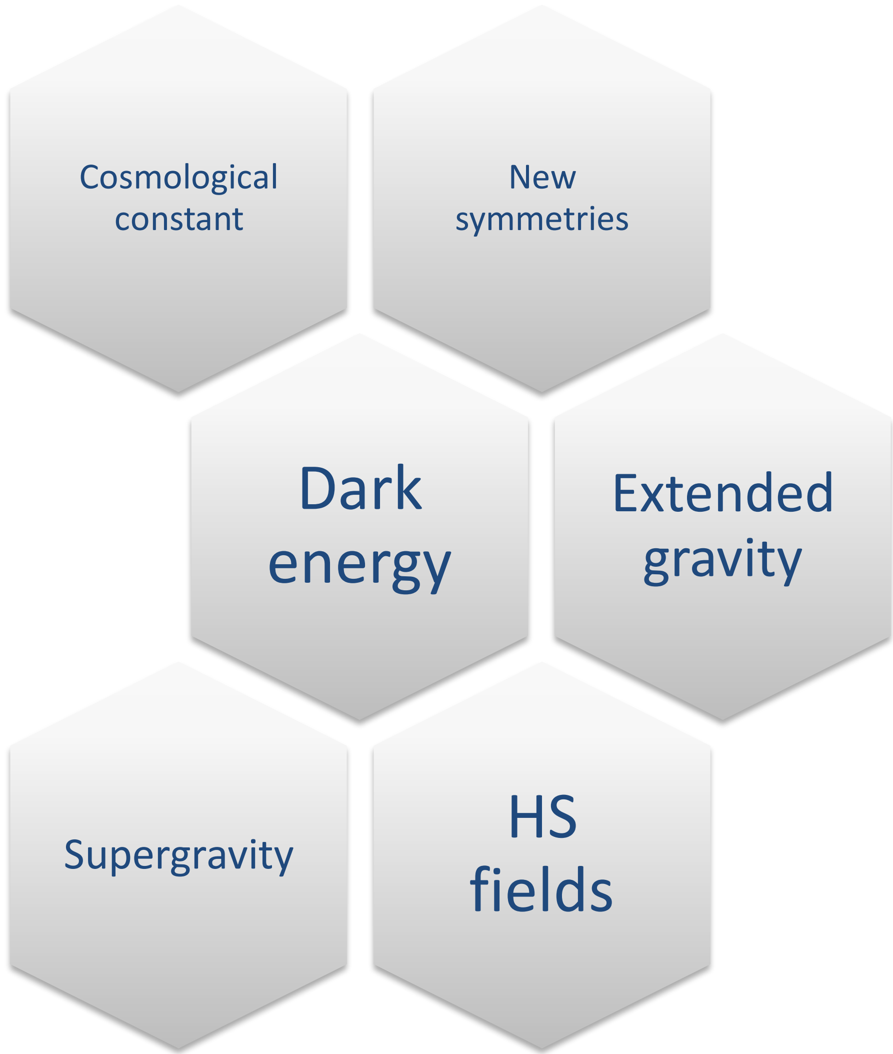
Gauge Theories of Some Important Groups

Lorentz group	Poincare group	Weyl group	De Sitter group	Affine group
R. Utiyama – 1956	T.W.B. Kibble – 1961	J.M. Charap and W. Tait – 1974	T. Kawai and H. Yoshida – 1979	A.B. Borisov and V. I. Ogievetskii – 1974

Gauge Theories of Maxwell Groups

Simple Maxwell Group	Semisimple Maxwell Group	Maxwell-Weyl Group	AdS-Maxwell Group	Maxwell-Affine Group
J.A. Azcarraga, K. Kamimura, J. Lukierski - 2010	D.V. Soroka, V.A. Soroka - 2011	O. Cebecioğlu, S. Kibaroğlu -2014	R. Durka, J. Kowalski-Glikman, M. Szczachor – 2011	O. Cebecioğlu, S. Kibaroğlu -2015

Influence Area of Maxwell Group



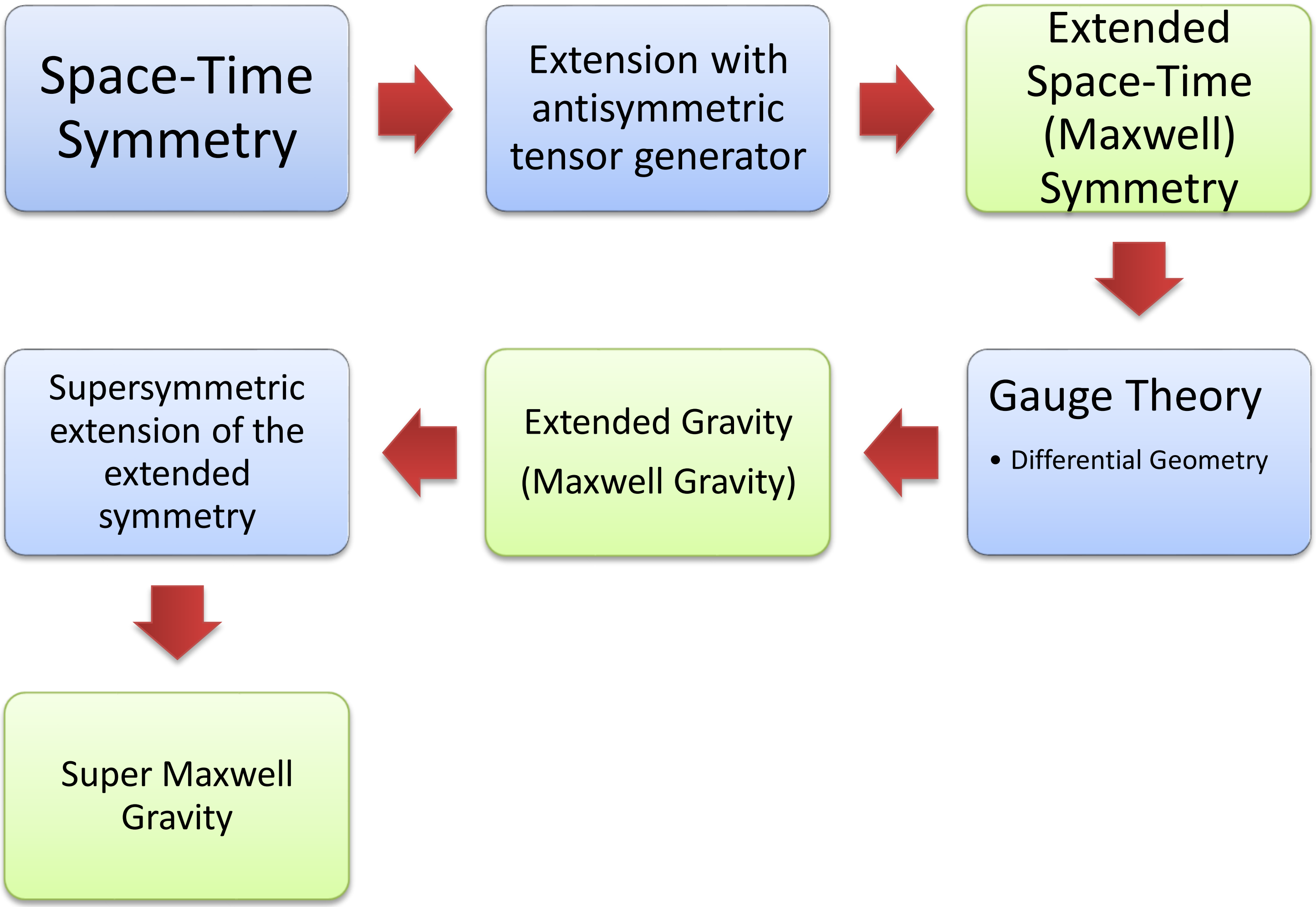
GENERAL OVERVIEW

1. Aim and method

Aim

Our fundamental aim is to going beyond the basic gravity.

Method


Space-Time Symmetry
Extension with antisymmetric tensor generator
Extended Space-Time (Maxwell) Symmetry
Gauge Theory
• Differential Geometry
Extended Gravity (Maxwell Gravity)
Supersymmetric extension of the extended symmetry
Super Maxwell Gravity

MAXWELL-WEYL GROUP (MW(1,3))

1. Tensor extension of Weyl algebra
2. Gauge theory of Maxwell-Weyl group

Tensor Extension of Weyl Algebra

Weyl algebra:

$$[M_{ab}, M_{cd}] = i(\eta_{ad}M_{bc} + \eta_{bc}M_{ad} - \eta_{ac}M_{bd} - \eta_{bd}M_{ac})$$
$$[M_{ab}, P_c] = i(\eta_{bc}P_a - \eta_{ac}P_b)$$
$$[P_a, P_b] = 0$$
$$[D, D] = 0$$
$$[P_a, D] = iP_a$$
$$[M_{ab}, D] = 0$$

Weyl algebra contains three generators (M_{ab}, P_a, D). These genrators correspond to Lorentz, momentum and scale symmetry respectively.

Differential realisation of the generators:

$$P_a = i\partial_a$$
$$D = i(x \cdot \partial)$$
$$M_{ab} = i(x_a\partial_b - x_b\partial_a)$$

Tensor Extension of Weyl Algebra

Maxwell-Weyl algebra:

$$[M_{ab}, M_{cd}] = i(\eta_{ad}M_{bc} + \eta_{bc}M_{ad} - \eta_{ac}M_{bd} - \eta_{bd}M_{ac})$$
$$[M_{ab}, P_c] = i(\eta_{bc}P_a - \eta_{ac}P_b)$$
$$[P_a, P_b] = iZ_{ab}$$
$$[D, D] = 0$$
$$[P_a, D] = iP_a$$
$$[M_{ab}, D] = 0$$
$$[M_{ab}, Z_{cd}] = i(\eta_{ad}Z_{bc} + \eta_{bc}Z_{ad} - \eta_{ac}Z_{bd} - \eta_{bd}Z_{ac})$$
$$[Z_{ab}, Z_{cd}] = 0$$
$$[Z_{ab}, P_c] = 0$$
$$[Z_{ab}, D] = 2iZ_{ab}$$

Weyl algebra can be extended as in the box.

This algebra satisfy all Jacobi Identities.

Where Z_{ab} are antisymmetric generators.

One can find this algebra with different notation in following paper:
S. Bonanos, J. Gomis, K. Kamimura, and J. Lukierski, Journal of Math. Phys. **51**, 102301 (2010)

Tensor Extension of Weyl Algebra

In order to find differential realisation of generators, we use following coset transformation;

$$K(x',\theta',\sigma')h(\omega) = g'(a,\varepsilon,\lambda,u)K(x,\theta,\sigma)$$

Where,

$$g(x,\theta,\sigma,\omega) = e^{ix(x)\cdot P}e^{i\theta(x)\cdot Z}e^{i\sigma(x)\cdot D}e^{-\frac{i}{2}\omega(x)\cdot M}$$

$$g(a,\varepsilon,\lambda,u) = e^{ia\cdot P}e^{i\varepsilon\cdot Z}e^{i\lambda\cdot D}e^{-\frac{i}{2}u\cdot M}$$

$$K(x,\theta,\sigma) = e^{ix(x)\cdot P}e^{i\theta(x)\cdot Z}e^{i\sigma(x)\cdot D}$$

$$\delta x^a = a^a + u^a{}_b x^b + \lambda x^a$$

$$\delta\theta^{ab} = \varepsilon^{ab} + u^{[a}{}_c\theta^{cb]} + 2\lambda\theta^{ab} - \tfrac{1}{4}a^{[a}x^{b]}$$

$$\delta\sigma = \lambda$$

$$\delta\omega^{ab} = u^{ab}$$

Tensor Extension of Weyl Algebra

Trasformation of scalar field is defined by;

$$\Phi'\left(x^a,\theta^{ab}\right)=\Phi\left(x^a-\delta x^a,\theta^{ab}-\delta\theta^{ab}\right)$$

If we use δx and $\delta\theta$ on the trasformation law of scalar field and compare following equation;

$$\delta\Phi\left(x,\theta\right)=\left(ia\cdot P+i\varepsilon\cdot Z+i\lambda D-\tfrac{i}{2}u\cdot M\right)\Phi\left(x,\theta\right)$$

Then we get;

$$P_a=i(\partial_a-\tfrac{1}{2}x^b\partial_{ab})$$

$$Z_{ab}=i\partial_{ab}$$

$$D=i\left(x^a\partial_a+2\theta^{ab}\partial_{ab}\right)$$

$$M_{ab}=i(x_a\partial_b-x_b\partial_a)+2i(\theta_a{}^c\partial_{bc}-\theta_b{}^c\partial_{ac})$$

Gauge Theory of Maxwell-Weyl Group

Defining gauge field ($\mathbb{A}$) by;

$$
\begin{aligned}
\mathbb{A} &= \mathbb{A}^A X_A \\
&= e^a P_a + B^{ab} Z_{ab} + \chi D - \tfrac{1}{2}\omega^{ab} M_{ab}
\end{aligned}
$$

Where X^A represents group generators and e^a, B^{ab}, χ, ω^{ab} are associated gauge fields.

If we going over the space-time indices e^a, B^{ab}, χ *and* ω^{ab} take the following form;

$$e^a = e^a_\mu dx^\mu,\ \omega^{ab} = \omega^{ab}_\mu dx^\mu,\ \chi = \chi_\mu dx^\mu,\ B^{ab} = B^{ab}_\mu dx^\mu$$

Also gauge field and its variation can be written by;

$$
\begin{aligned}
\mathbb{A}_\mu &= e^a_\mu P_a + B^{ab}_\mu Z_{ab} + \chi_\mu D - \tfrac{1}{2}\omega^{ab}_\mu M_{ab} \\
\delta\mathbb{A}_\mu &= \delta e^a_\mu P_a + \delta B^{ab}_\mu Z_{ab} + \delta\chi_\mu D - \tfrac{1}{2}\delta\omega^{ab}_\mu M_{ab}
\end{aligned}
$$

Gauge Theory of Maxwell-Weyl Group

To find variation of gauge field, we can use the following formula;

$$\delta \mathcal{A}_\mu = -\partial_\mu \zeta - i\left[\mathcal{A}_\mu, \zeta\right]$$

Under a local gauge transformation with the values in Maxwell algebra $\zeta(x)$;

$$\begin{aligned}\zeta(x) &= \zeta^A(x) X_A \\ &= y^a(x) P_a + \varphi^{ab}(x) Z_{ab} + \rho(x) D - \tfrac{1}{2}\tau^{ab}(x) M_{ab}\end{aligned}$$

We get following transformations;

$$\delta e^a_\mu = -\partial_\mu y^a - \omega^a_{\mu\ c} y^c - \chi_\mu y^a + e^c_\mu \tau^a{}_c + \rho e^a_\mu$$

$$\delta B^{ab}_\mu = -\partial_\mu \varphi^{ab} - \omega^{[a}_{\mu\ c}\varphi^{cb]} - 2\chi_\mu \phi^{ab} + \tau^{[a}{}_c B^{cb]}_\mu + 2\rho B^{ab}_\mu + \frac{1}{2} e^{[a}_\mu y^{b]}$$

$$\delta \chi_\mu = -\partial_\mu \rho$$

$$\delta \omega^{ab}_\mu = -\partial_\mu \tau^{ab} - \omega^{[a}_{\mu\ c}\tau^{cb]}$$

Gauge Theory of Maxwell-Weyl Group

The curvature forms can be found with the use of following equations;

$$\mathbb{F} = d\mathbb{A} + \tfrac{i}{2}\left[\mathbb{A}, \mathbb{A}\right]$$

Where:

$$\mathbb{F} = \mathbb{F}^{A} X_{A} = F^{a} P_{a} + F^{ab} Z_{ab} + fD - \tfrac{1}{2} R^{ab} M_{ab}$$

One gets;

$$F^{a} = de^{a} + \omega^{a}{}_{b} \wedge e^{b} + \chi \wedge e^{a}$$

$$F^{ab} = dB^{ab} + \omega^{[a}{}_{c} \wedge B^{cb]} + 2\chi \wedge B^{ab} - \tfrac{1}{2} e^{a} \wedge e^{b}$$

$$f = d\chi$$

$$R^{ab} = d\omega^{ab} + \omega^{a}{}_{c} \wedge \omega^{cb}$$

If we going over space-time indices;

$$F^{a} = \tfrac{1}{2} F^{a}_{\mu\upsilon} dx^{\mu} \wedge dx^{\upsilon}$$

$$F^{ab} = \tfrac{1}{2} F^{ab}_{\mu\upsilon} dx^{\mu} \wedge dx^{\upsilon}$$

$$f = \tfrac{1}{2} f_{\mu\upsilon} dx^{\mu} \wedge dx^{\upsilon}$$

$$R^{ab} = \tfrac{1}{2} R^{ab}_{\mu\upsilon} dx^{\mu} \wedge dx^{\upsilon}$$

$$F^{a}_{\mu\upsilon} = \partial_{[\mu} e^{a}_{\upsilon]} + \omega^{a}_{[\mu b} e^{b}_{\upsilon]} + \chi_{[\mu} e^{a}_{\upsilon]}$$

$$F^{ab}_{\mu\upsilon} = \partial_{[\mu} B^{ab}_{\upsilon]} + \omega^{[a}_{[\mu|c} B^{cb]}_{\upsilon]} + 2\chi_{[\mu} B^{ab}_{\upsilon]} - \frac{1}{2} e^{a}_{[\mu} e^{b}_{\upsilon]}$$

$$f_{\mu\upsilon} = \partial_{[\mu} \chi_{\upsilon]}$$

$$R^{ab}_{\mu\upsilon} = \partial_{[\mu} \omega^{ab}_{\upsilon]} + \omega^{a}_{[\mu|c} \omega^{cb}_{\upsilon]}$$

Gauge Theory of Maxwell-Weyl Group

To find the variation of curvatures under local gauge transformation one uses;

$$\delta \mathbb{F}_{\mu\upsilon} = i\left[\zeta, \mathbb{F}_{\mu\upsilon}\right]$$

We get;

$$\delta F^{a}_{\mu\upsilon} = -y^{a} f_{\mu\upsilon} - R^{a}_{\mu\upsilon\ b} y^{b} + \rho F^{a}_{\mu\upsilon} + \tau^{a}{}_{b} F^{b}_{\mu\upsilon}$$

$$\delta F^{ab}_{\mu\upsilon} = -2\varphi^{ab} F_{\mu\upsilon} + \varphi^{[a}{}_{c} R^{cb]}_{\mu\upsilon} - \frac{1}{2} y^{[a} F^{b]}_{\mu\upsilon} + 2\rho F^{ab}_{\mu\upsilon} + \tau^{[a}{}_{c} F^{cb]}_{\mu\upsilon}$$

$$\delta f_{\mu\upsilon} = 0$$

$$\delta R^{ab}_{\mu\upsilon} = \tau^{[a}{}_{c} R^{cb]}_{\mu\upsilon}$$

Gauge Theory of Maxwell-Weyl Group

To find an invariant Lagrangian under gauge transformation, we have to take into consideration scale(dilatation) symmetry.

In our case, variation of metric tensor does not vanish. This issue relates to Weyl gauge theory.

$$\delta g_{\mu\nu}(x) = 2\rho(x) g_{\mu\nu}(x)$$

This is major difficulty of constructing of an invariant Lagrangian.

Describing a weyl weight by ‘w(f)’ then one gets expressions about metric.

$$w(g_{\mu\nu}) = +2$$

$$w(g^{\mu\nu}) = -2$$

$$w\left(\sqrt{g_{\mu\nu}}\right) = w(e) = +4$$

$$S_f = \int d^4x e \mathcal{L}_f$$

The free gravitational action requires Weyl weight zero. This condition implies that the Lagrangian density must have $w(L_f) = -4$.

Gauge Theory of Maxwell-Weyl Group

Weyl weights of curvatures and gauge fields written as;

$w\left(F^{a}_{\mu\nu}\right)=1$	$w\left(e^{a}_{\mu\nu}\right)=1$
$w\left(F^{ab}_{\mu\nu}\right)=2$	$w\left(B^{ab}_{\mu\nu}\right)=2$
$w\left(f_{\mu\nu}\right)=0$	$w\left(\chi_{\mu\nu}\right)=0$
$w\left(R^{ab}_{\mu\nu}\right)=0$	$w\left(\omega^{ab}_{\mu\nu}\right)=0$
$w\left(R\right)=-2$	$w\left(e\right)=4$

One can easily see that the Einstein-Hilbert action does not invariant under the scale transformation.

$$S_{E-H}=\frac{1}{2\kappa}\int d^4xeR$$

To overcome this difficulty, if we multiply it by a compensating scalar field ϕ introduced by Brans-Dicke (1961) and elaborated by Dirac (1973), we can form a Weyl invariant action linear in R.

In our approach we will follow Dirac's idea. The scalar field ϕ with $w(\phi)=-1$ lets $\phi^2 R$ be a regular part of L_f, and hence the combination is invariant under scale transformation.

$$\delta\phi=-\rho\phi$$

$$S=\frac{1}{2\kappa}\int d^4xe\phi^2R$$

C. Brans and R. H. Dicke, Phys. Rev. **124**, 925 (1961)
P. A. M. Dirac, Proc. R. Soc. Lon. A. 333, 403-418 (1973)

Gauge Theory of Maxwell-Weyl Group

Covariant derivative can be written following form;

$$\mathcal{D}\Phi = \left[d + \omega + w(\Phi)\chi \right]\Phi$$

The curvatures take form;

$$F^a = \mathcal{D}e^a$$

$$F^{ab} = \mathcal{D}B^{ab} - \tfrac{1}{2} e^a \wedge e^b$$

$$f = d\chi$$

$$R^{ab} = \mathcal{D}\omega^{ab}$$

From here one can write Bianchi identities;

$$\mathcal{D}F^a = R^a{}_b \wedge e^b + f \wedge e^a,$$

$$\mathcal{D}F^{ab} = R^{[a}{}_c \wedge B^{cb]} + 2f \wedge B^{ab} - \frac{1}{2} F^{[a} \wedge e^{b]},$$

$$\mathcal{D}R^{ab} = 0, \quad \mathcal{D}f = 0.$$

Now we can start to construct Lagrangian.

Gauge Theory of Maxwell-Weyl Group

It is easy to see that following equation has zero weyl weight. We can start to construct Lagrangian with this shifted curvature;

$$\mathcal{J}^{ab} = R^{ab} + 2\gamma\phi^2 F^{ab}$$

With this combination we have an Einstein Lagrangian that involves the curvature scalar linearly. Therefore we consider the following Lagrangian density 4-form as our starting point for the free gravitational part:

$$\mathcal{L}_f = \frac{1}{2\kappa\gamma} J \wedge {}^*J = \frac{1}{4\kappa\gamma}\varepsilon_{abcd} J^{ab} \wedge J^{cd}$$
$$= \frac{1}{4\kappa\gamma}\varepsilon_{abcd} R^{ab} \wedge R^{cd} + \phi^2 \frac{1}{\kappa}\varepsilon_{abcd} R^{ab} \wedge F^{cd} + \phi^4 \frac{\gamma}{\kappa}\varepsilon_{abcd} F^{ab} \wedge F^{cd}$$

Where γ and κ are constants and the first term can be ignored because it is a closed form.

Gauge Theory of Maxwell-Weyl Group

The introduction of a compensating field forces us to add its kinetic term to the Lagrangian. We then get the total action for vacuum as follows:

$$\mathcal{L}_0 = f \wedge f - \frac{1}{2}\mathcal{D}\phi \wedge {}^*\mathcal{D}\phi + \frac{\lambda}{4}\phi^4\,{}^*1$$

Where λ is another constant. Our complete action is the sum of the free gravity action and the vacuum action,

$$S = \int \left(\frac{1}{4\kappa\gamma}\varepsilon_{abcd}R^{ab} \wedge R^{cd} + \phi^2 \frac{1}{\kappa}\varepsilon_{abcd}R^{ab} \wedge F^{cd} + \phi^4 \frac{\gamma}{\kappa}\varepsilon_{abcd}F^{ab} \wedge F^{cd} + f \wedge f - \frac{1}{2}\mathcal{D}\phi \wedge {}^*\mathcal{D}\phi + \frac{\lambda}{4}\phi^4\,{}^*1 \right)$$

Gauge Theory of Maxwell-Weyl Group

Then we get following equations of motion;

$$\delta_\omega S = 0 \rightarrow \mathcal{D}\left(\phi^2 F^{ab}\right) - \mathcal{J}^{[a}{}_c \wedge B^{cb]} = 0$$

$$\delta_e S = 0 \rightarrow \left(\begin{aligned} & -\phi^2 \frac{1}{\kappa} \varepsilon_{abcd} \mathcal{J}^{ab} \wedge e^d + \frac{1}{2}\left[\mathcal{D}_c \phi\, {}^*\mathcal{D}\phi + \mathcal{D}\phi \wedge {}^*\left(e_a \wedge e_c\right) \mathcal{D}^a \phi \right] \\ & -\frac{1}{4}\left(f_{cb} e^b \wedge {}^*f - \frac{1}{2}\varepsilon_{abcd} f^{ab} e^d \wedge f \right) + \frac{\lambda}{4}\phi^4\, {}^*e_c \end{aligned} \right) = 0$$

$$\delta_B S = 0 \rightarrow \mathcal{D}\left(\phi^2 \mathcal{J}^{ab}\right) = 0$$

$$\delta_\chi S = 0 \rightarrow \frac{2\phi^2}{\kappa} \varepsilon_{abcd} \mathcal{J}^{ab} \wedge B^{cd} + \frac{1}{2}\mathcal{D}\, {}^*f + \phi\, {}^*\mathcal{D}\phi = 0$$

$$\delta_\phi S = 0 \rightarrow \frac{2\phi}{\kappa} \varepsilon_{abcd} \mathcal{J}^{ab} \wedge F^{cd} + \mathcal{D}\, {}^*\mathcal{D}\phi + \lambda \phi^3\, {}^*1 = 0$$

Gauge Theory of Maxwell-Weyl Group

One can find following expression thanks to using second equation of previous page;

$$\phi^2\left(\mathcal{J}^a{}_b-\frac{1}{2}\delta^a{}_b\mathcal{J}\right)=-\frac{\kappa}{2}\left[\mathcal{D}^a\phi\mathcal{D}_b\phi-\frac{1}{2}\delta^a{}_b\left(\mathcal{D}^c\phi\mathcal{D}_c\phi-\frac{\lambda}{2}\phi^4\right)+\left(f^{ac}f_{cb}-\frac{1}{4}\delta^a{}_bf^{cd}f_{cd}\right)\right]$$

If we swich from tangent space indices to space-time indices then one gets the field equation with a cosmological term depending on the dilaton field;

$$R^\mu{}_\alpha-\frac{1}{2}\delta^\mu{}_\alpha R-3\gamma\phi^2\delta^\mu{}_\alpha=2\gamma T(B)^\mu{}_\alpha-\frac{\kappa}{2}\phi^{-2}\left[T(\phi)^\mu{}_\alpha+T(f)^\mu{}_\alpha\right]$$

The energy-momentum tensors:

$$T(B)^\mu{}_\alpha=e^\mu_ae^\beta_b\mathcal{D}_{[\alpha}B^{ab}_{\beta]}-\frac{1}{2}\delta^\mu{}_\alpha\left(e^\rho_ae^\sigma_b\mathcal{D}_{[\rho}B^{ab}_{\sigma]}\right)$$

$$T(\phi)^\mu{}_\alpha=\mathcal{D}^\mu\phi\mathcal{D}_\alpha\phi-\frac{1}{2}\delta^\mu{}_\alpha\left(\mathcal{D}^\gamma\phi\mathcal{D}_\gamma\phi-\frac{\lambda}{2}\phi^4\right)$$

$$T(f)^\mu{}_\alpha=f^{\mu\beta}f_{\beta\alpha}-\frac{1}{4}\delta^\mu{}_\alpha f^{\gamma\delta}f_{\gamma\delta}$$

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Thank you