Magnetic monopoles in noncommutative quantum mechanics

Samuel Kováčik

Commenius University Bratislava
(soon) Dublin Institute for Advanced Studies

arXiv:1604.05968

25.8.2016

Magnetic monopoles

Magnetic monopoles

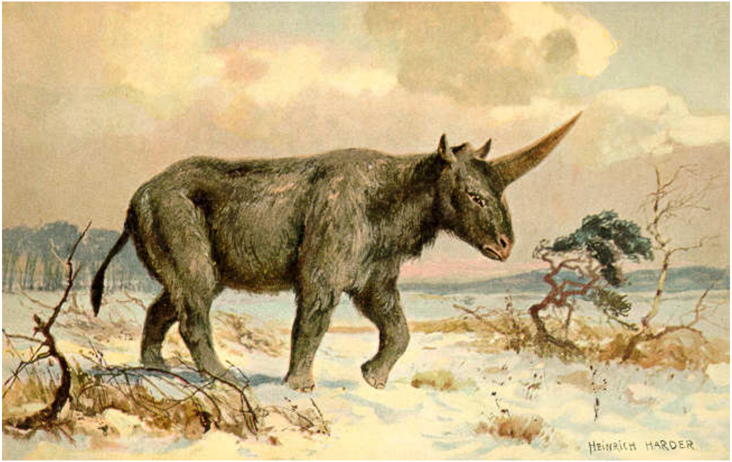
$\#$ of observed magnetic monopoles $\approx \#$ of observed unicorns

Magnetic monopoles

$\#$ of observed magnetic monopoles $<$ $\#$ of observed unicorns

Magnetic monopoles

Figure: *Elasmotherium sibiricum*, Giant Siberian Unicorn, extinct



Magnetic monopoles

Maxwell's equations

$$
\begin{aligned}
\text{div } \vec{E}(\vec{r},t) &= 4\pi\rho_E(\vec{r},t)\,, \\
\text{div } \vec{B}(\vec{r},t) &= 4\pi\rho_M(\vec{r},t)\,, \\
\text{rot } \vec{E}(\vec{r},t) &= -\frac{\partial \vec{B}(\vec{r},t)}{\partial t} - 4\pi\vec{J}_M(\vec{r},t)\,, \\
\text{rot } \vec{B}(\vec{r},t) &= \frac{\partial \vec{E}(\vec{r},t)}{\partial t} + 4\pi\vec{J}_E(\vec{r},t)\,.
\end{aligned}
$$

Magnetic monopoles

Maxwell's equations

$$\begin{aligned}
\text{div } \vec{E}(\vec{r},t) &= 4\pi\rho_E(\vec{r},t)\,,\\
\text{div } \vec{B}(\vec{r},t) &= 4\pi\rho_M(\vec{r},t)\,,\\
\text{rot } \vec{E}(\vec{r},t) &= -\frac{\partial\vec{B}(\vec{r},t)}{\partial t} - 4\pi\vec{J}_M(\vec{r},t)\,,\\
\text{rot } \vec{B}(\vec{r},t) &= \frac{\partial\vec{E}(\vec{r},t)}{\partial t} + 4\pi\vec{J}_E(\vec{r},t)\,.
\end{aligned}$$

Ordinary QM

$$\begin{aligned}
\vec{L} &= \vec{r}\times\vec{v} - \mu\frac{\vec{r}}{r},\\
\text{DQC: } \mu &= eg \in \mathbb{Z}/2,\\
[\pi_i,\pi_j] &= i\mu\varepsilon^{ijk}\frac{\vec{r}}{r^3},\ \pi \text{ are canonical momenta,}
\end{aligned}$$

Magnetic monopoles

One of the safest bets that one can make about physics not yet seen,
Polchinski 2012

NC space

Three dimensional rotationally invariant NC space defined by $\lambda \approx l_{\text{Planck}}$.

R^3_λ

- $x_i = \lambda \sigma^i_{\alpha\beta} a^+_\alpha a_\beta$, $r = \lambda(a^+_\alpha a_\alpha + 1)$; $\alpha, \beta = 1, 2$
  where σ^i are the Pauli matrices and the c/a operators satisfy

$$[a_\alpha, a^+_\beta] = \delta_{\alpha\beta}, \ [a_\alpha, a_\beta] = [a^+_\alpha, a^+_\beta] = 0 \, , |n_1, n_2\rangle = \frac{(a^+_1)^{n_1} \, (a^+_2)^{n_2}}{\sqrt{n_1! \, n_2!}} \, |0\rangle$$

Physical states

Hilbert space $\mathcal{H}_\lambda$

- Spanned on analytic functions equipped with a norm

$$\Psi \;=\; \sum C(m,n)\, a_1^{+m_1}\, a_2^{+m_2}\, a_1^{n_1}\, a_2^{n_2}, \;\; \|\Psi\|^2 \;=\; 4\pi\,\lambda^2\,\mathrm{Tr}[\hat{r}\,\Psi^\dagger\,\Psi].$$

Special attention is paid to $\Psi_{\kappa jm}$

$$\kappa = m_1 + m_2 - n_1 - n_2.$$

Operators in $\mathcal{H}$

- $\hat{H}_0\Psi = \frac{1}{2m\lambda r}\,[a_\alpha^+, [a_\alpha, \Psi]]$
- $\hat{L}_i\Psi = \frac{1}{2\lambda}[x_i, \Psi], \;\; [\hat{L}_i, \hat{L}_j] = i\varepsilon^{ijk}\hat{L}_k$
- $\hat{X}_i\Psi = \frac{1}{2}(x_i\Psi + \Psi x_i), \;\; \hat{r}\Psi = \frac{1}{2}\,(r\Psi + \Psi r)$

Monopole states $\leftrightarrow$ generalized states

$$\Psi_\kappa(e^{-i\tau}a^+, e^{i\tau}a) = e^{-i\tau\kappa}\Psi_\kappa(a^+, a),\ \tau \in \mathbf{R},\ \text{fixed } \kappa \in \mathbb{Z},$$

Comparison MM vs κ states

$$[\hat{x}_i, \hat{x}_j] = 0 \quad \leftrightarrow \quad [\hat{X}_i, \hat{X}_j] = \lambda^2 \varepsilon_{ijk} \hat{L}_k,$$

$$[\hat{x}_i, \hat{\pi}_j] = i\delta_{ij} \quad \leftrightarrow \quad [\hat{X}_i, \hat{V}_j] = i\delta_{ij}\left(1 - \lambda^2 \hat{H}_0\right),$$

$$[\hat{\pi}_i, \hat{\pi}_j] = i\mu\varepsilon_{ijk}\frac{\hat{x}_k}{r^3} \quad \leftrightarrow \quad \left[\hat{V}_i, \hat{V}_j\right] = i\frac{-\kappa}{2}\varepsilon_{ijk}\frac{\hat{X}_k}{\hat{r}(\hat{r}^2 - \lambda^2)}.$$

$$\hat{C}_1 = -q\mu \quad \leftrightarrow \quad \hat{C}_1 = \frac{\kappa}{2}q,$$

$$\hat{C}_2 = q^2 + (\mu)^2(-2E) \quad \leftrightarrow \quad \hat{C}_2 = q^2 + \left(\frac{\kappa}{2}\right)^2(-2E + \lambda^2 E^2).$$

Monopole states $\leftrightarrow$ generalized states

$$\Psi_\kappa(e^{-i\tau}a^+, e^{i\tau}a) = e^{-i\tau\kappa}\Psi_\kappa(a^+, a),\ \tau \in \mathbf{R},\ \text{fixed } \kappa \in \mathbb{Z},$$

Comparison MM vs κ states

$$\begin{aligned}
[\hat{x}_i, \hat{x}_j] = 0 \quad &\leftrightarrow \quad [\hat{X}_i, \hat{X}_j] = \lambda^2 \varepsilon_{ijk} \hat{L}_k, \\
[\hat{x}_i, \hat{\pi}_j] = i\delta_{ij} \quad &\leftrightarrow \quad [\hat{X}_i, \hat{V}_j] = i\delta_{ij}\left(1 - \lambda^2 \hat{H}_0\right), \\
[\hat{\pi}_i, \hat{\pi}_j] = i\mu\varepsilon_{ijk}\frac{\hat{x}_k}{r^3} \quad &\leftrightarrow \quad \left[\hat{V}_i, \hat{V}_j\right] = i\frac{-\kappa}{2}\varepsilon_{ijk}\frac{\hat{X}_k}{\hat{r}(\hat{r}^2 - \lambda^2)}. \\
\hat{C}_1 = -q\mu \quad &\leftrightarrow \quad \hat{C}_1 = \frac{\kappa}{2}q, \\
\hat{C}_2 = q^2 + (\mu)^2(-2E) \quad &\leftrightarrow \quad \hat{C}_2 = q^2 + \left(\frac{\kappa}{2}\right)^2(-2E + \lambda^2 E^2).
\end{aligned}$$

$$\mu \in \mathbb{Z}/2 \leftrightarrow -\kappa/2 \in \mathbb{Z}/2.$$

Thank you for your attention.